Deep Generative Models: Image Editing with Diffusion Models

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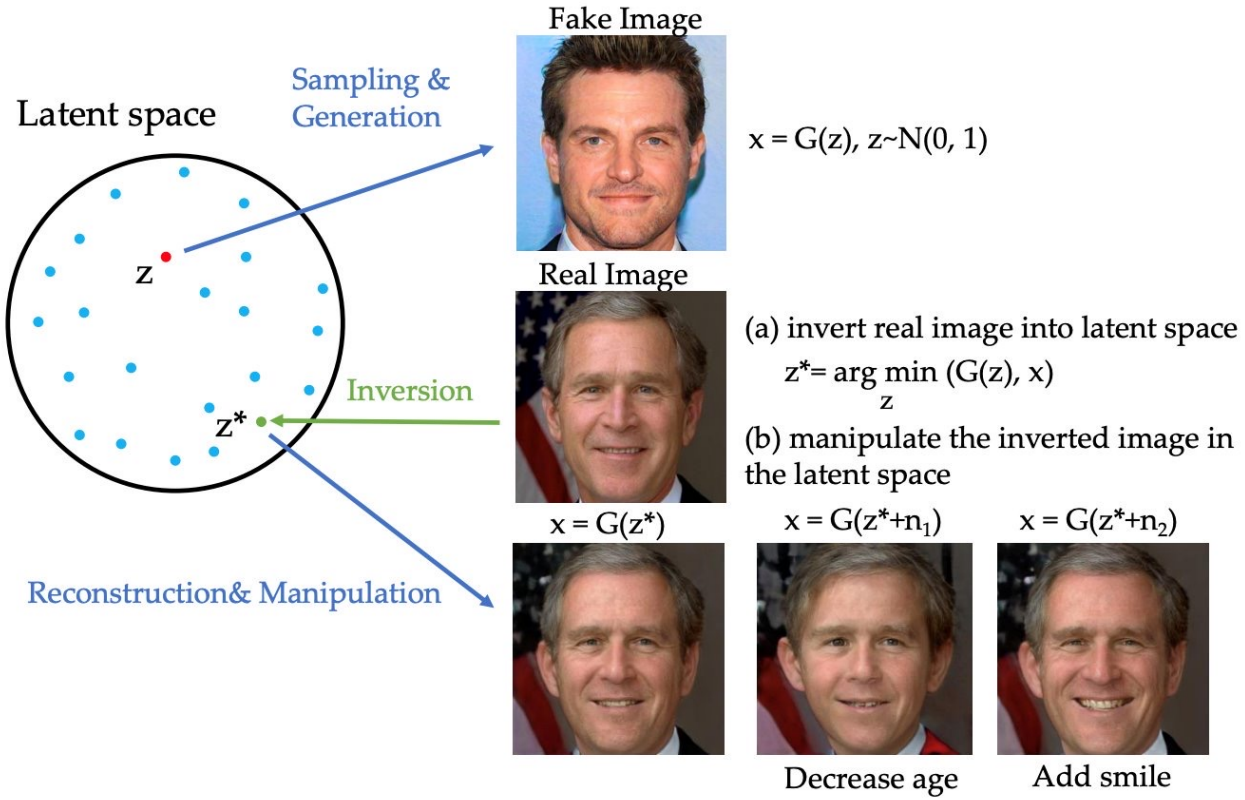
Outline

- Markov Hierarchical Variational Auto Encoders (MHVAEs)
  - Autoregressive Encoder and Autoregressive Decoder of an MHVAE
  - Derivation of the ELBO of an MHVAE
- Diffusion Models: MHVAEs with a Linear Gaussian Autoregressive Latent Space
  - Forward Diffusion Process
  - Reverse Diffusion Process
  - ELBO for Diffusion Models as a particular case of the ELBO for MHVAEs
- Implementation Details: UNet architecture, Training and Sampling Strategies
- Application of Diffusion Models
  - Text-Conditioned Diffusion Model: Stable Diffusion
  - Multimodal Control for Consistent Synthesis: ControlNet
  - **Image Editing: DDIM, P2P**

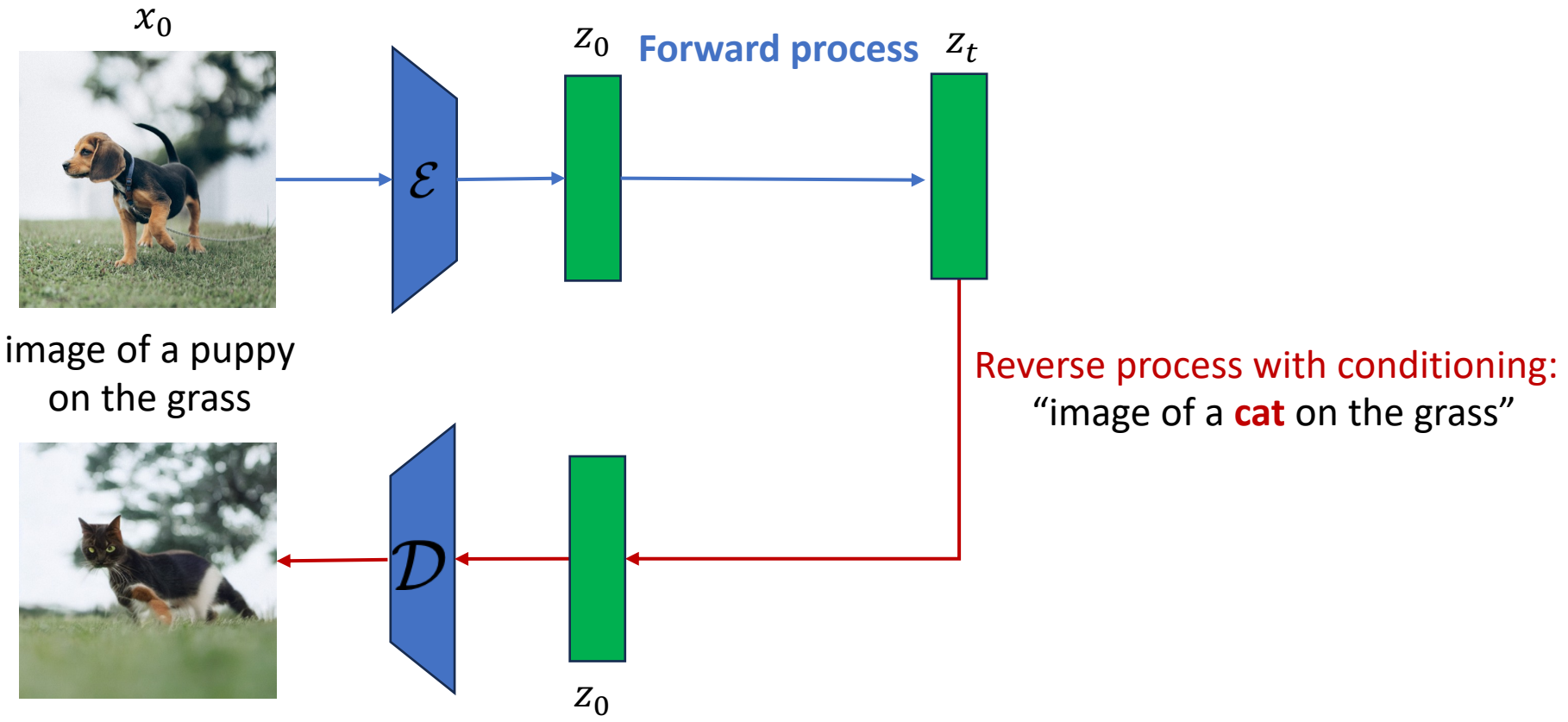
Latent Space Image Editing: Inversion + Manipulation

- Diffusion models so far can be used for image generation.
- Stable Diffusion performs text-to-image conditioning in a rich latent space.
- Can we use the latent space of diffusion models to perform image editing?



Naïve Image Editing Idea

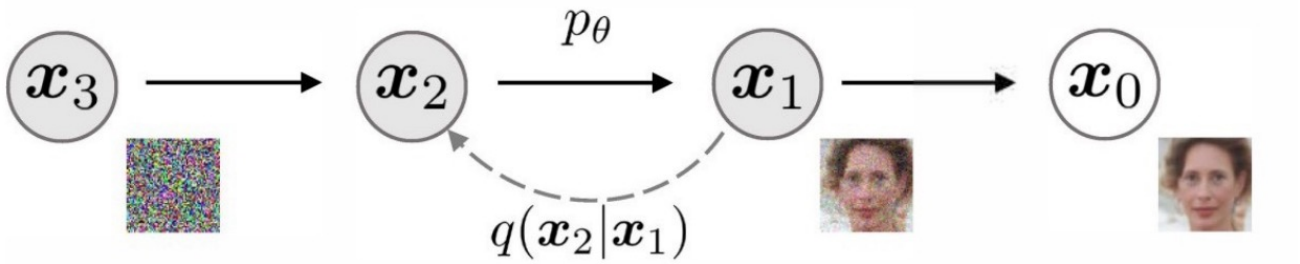
- Instead of starting from pure noise, let us perform naïve inversion using the forward process and a fixed image.



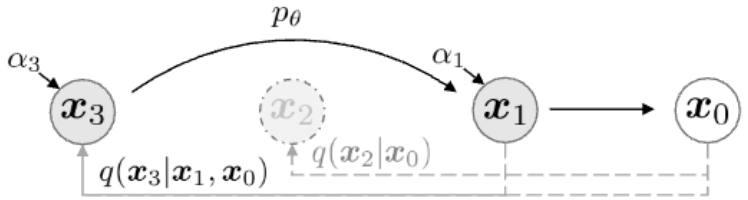
- Depending on how much noise is added, we can change a lot of features in the image or not enough features.

How to improve the inversion?

- Problems
  - Randomness in the model: if we encode x_0 to x_t using the forward process and then run the reverse process, we will not get x_0.
  - The reverse process requires T sequential steps, which can be slow.



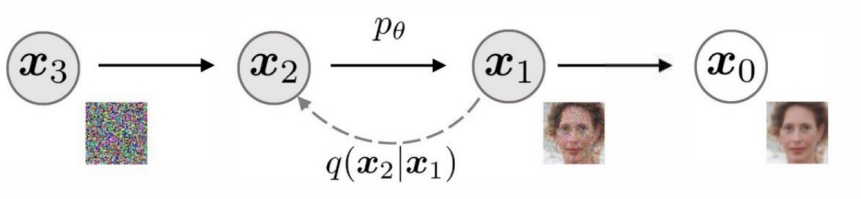
- What if we had a different sampling mechanism?

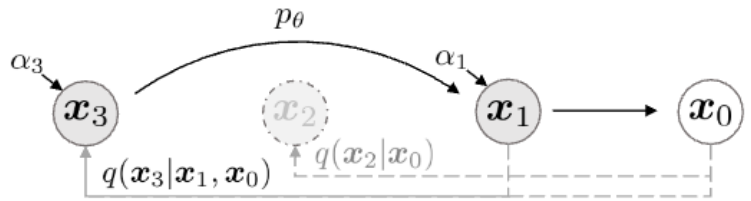


- We will introduce a sampling process that allows for better inversion and image editing.

Designing a Faster Processes

- In diffusion models, the reverse process is designed to approximate the forward process.
- Intuition: If we had a forward process with few steps, the backward process would also require a small number of steps to sample a new image.
- How can we design sampling processes with fewer steps?





- We will generalize the Markovian forward process of DDPM to non-Markovian processes to obtain a large family of models.
- Then, we can select a diffusion process that can be simulated in few steps to achieve fast sampling!

Recall the Forward Process of DDPM

$$\overline{\alpha_t} = \prod_{i=1}^{t} \alpha_i$$

- Recall that the forward process of DDPM is Markovian and given by

$$q(x_t \mid x_{t-1}) = \mathcal{N}(x_t; \sqrt{\alpha_t} x_{t-1}, (1-\alpha_t) I)$$

$$q(x_t \mid x_0) = \mathcal{N}\left(x_t; \sqrt{\overline{\alpha_t}} x_0, (1-\overline{\alpha_t}) I\right)$$

- Therefore, reversing q gives

$$q(x_{t-1} \mid x_t, x_0) = \frac{q(x_t \mid x_{t-1}, x_0) q(x_{t-1} \mid x_0)}{q(x_t \mid x_0)}$$

$$= \frac{\mathcal{N}\left(x_t; \sqrt{\alpha_t} x_{t-1}, (1-\alpha_t) I\right) \mathcal{N}\left(x_{t-1}; \sqrt{\overline{\alpha_{t-1}}} x_0, (1-\overline{\alpha_{t-1}}) I\right)}{\mathcal{N}\left(x_t; \sqrt{\overline{\alpha_t}} x_0, (1-\overline{\alpha_t}) I\right)}$$

$$\propto \mathcal{N}\Big(x_{t-1}; \underbrace{\frac{\sqrt{\alpha_t}(1-\overline{\alpha_{t-1}}) x_t + \overline{\overline{\alpha}}_{t-1}(1-\alpha_t) x_0}{1-\overline{\alpha_t}}}_{\mu_q(x_t, x_0)}, \underbrace{\frac{(1-\alpha_t)(1-\overline{\alpha_{t-1}})}{1-\overline{\alpha_t}} I}_{\Sigma_q(t)}\Big)$$

Generalized Non-Markovian Processes

- Define the generalized posterior distribution

$$q_\sigma(x_{t-1} \mid x_t, x_0) = \mathcal{N}\left(\sqrt{\overline{\alpha_{t-1}}}x_0 + \sqrt{1-\overline{\alpha_{t-1}}-\sigma_t^2}\cdot\frac{x_t-\sqrt{\overline{\alpha_t}}x_0}{\sqrt{1-\overline{\alpha_t}}}, \sigma_t^2 I\right),$$

  where $\sigma_t \geq 0$ is a variance parameter.

- The generalized posterior q_σ is designed such that it maintains the same forward distribution $q(x_t|x_0)$ as in DDPM.

- Different choices of $\sigma_t \geq 0$ result in different generative models.
  - For $\sigma_t = \frac{(1-\alpha_t)(1-\overline{\alpha_{t-1}})}{1-\overline{\alpha_t}}$, we obtain DDPM.
  - For $\sigma_t = 0, \forall t \geq 0$, the process is deterministic!

- We will see that setting $\sigma_t = 0, \forall t \geq 0$, will allow for deterministic denoising and faster sampling!

ELBO for the Generalized Process

- Recall that the ELBO reduces to:

$$D_{\mathrm{KL}}\left(q(x_{t-1} \mid x_t, x_0) \,||\, p_\theta(x_{t-1} \mid x_t)\right) = D_{\mathrm{KL}}\left(\mathcal{N}\left(x_{t-1}; \mu_q(x_t, x_0), \Sigma_q\right) || \, \mathcal{N}\left(x_{t-1}; \mu_\theta(x_t, t), \Sigma_q\right)\right)$$

$$= \frac{1}{2\sigma_q^2(t)}\left[|| \, \mu_\theta - \mu_q \, ||_2^2\right]$$

- Choosing mean of $p_\theta(x_{t-1} \mid x_t)$ to match form of mean of $q(x_{t-1} \mid x_t, x_0)$
  - In DDPM we had:

$$\mu_q(x_t, x_0) = \frac{\sqrt{\alpha_t}(1-\overline{\alpha_{t-1}})x_t + \sqrt{\overline{\alpha_{t-1}}}(1-\alpha_t)x_0}{1-\overline{\alpha_t}} \quad \Rightarrow \quad \mu_\theta(x_t, t) = \frac{\sqrt{\alpha_t}(1-\overline{\alpha_{t-1}})x_t + \sqrt{\overline{\alpha_{t-1}}}(1-\alpha_t)\widehat{x_\theta}(x_t, t)}{1-\overline{\alpha_t}}$$

- For the generalized process we have:

$$\mu_q(x_t, x_0) = \sqrt{\overline{\alpha_{t-1}}}x_0 + \sqrt{1-\overline{\alpha_{t-1}}-\sigma_t^2}\,\frac{x_t - \sqrt{\overline{\alpha_t}}x_0}{\sqrt{1-\overline{\alpha_t}}} \quad \Rightarrow \quad \mu_\theta(x_t, t) = \sqrt{\overline{\alpha_{t-1}}}\widehat{x_\theta}(x_t, t) + \sqrt{1-\overline{\alpha_{t-1}}-\sigma_t^2}\,\frac{x_t - \sqrt{\overline{\alpha_t}}\widehat{x_\theta}(x_t, t)}{\sqrt{1-\overline{\alpha_t}}}$$

Reparameterization as an Alternative Form for ELBO

- Plugging $x_0 = \frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t,t)}{\sqrt{\overline{\alpha}_t}}$ into the denoising transition mean $\mu_q(x_t, x_0)$, we obtain:

$$\mu_q(x_t, x_0) = \sqrt{\overline{\alpha}_{t-1}}x_0 + \sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\,\frac{x_t - \sqrt{\overline{\alpha}_t}x_0}{\sqrt{1-\overline{\alpha}_t}}$$

$$= \sqrt{\overline{\alpha}_{t-1}}\,\frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t,t)}{\sqrt{\overline{\alpha}_t}} + \sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\,\frac{x_t - \sqrt{\overline{\alpha}_t}\,\frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t,t)}{\sqrt{\overline{\alpha}_t}}}{\sqrt{1-\overline{\alpha}_t}}$$

$$= \sqrt{\overline{\alpha}_{t-1}}\,\frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t,t)}{\sqrt{\overline{\alpha}_t}} + \sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\,\hat{\epsilon}_\theta(x_t,t)$$

- Choosing the mean and variance of p to match the mean and variance of q:

$$\mu_\theta(x_t,t) = \sqrt{\overline{\alpha}_{t-1}}\,\frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t,t)}{\sqrt{\overline{\alpha}_t}} + \sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\,\hat{\epsilon}_\theta(x_t,t), \qquad \Sigma_\theta(t) = \sigma_t^2 I$$

What have we achieved so far?

$$\mu_\theta(x_t, t) = \sqrt{\overline{\alpha}_{t-1}} \frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t, t)}{\sqrt{\overline{\alpha}_t}} + \sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\cdot\hat{\epsilon}_\theta(x_t, t)$$

- We created a new generalized inference distribution with the same training objective as in DDPM.
- The generalized process captures a rich family of generative processes depending on the selection of the parameter σ_t.
- We can select σ_t to achieve much faster sampling!

- Recall that $p_\theta(x_{t-1} \mid x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \sigma_t^2 I)$ and thus

$$x_{t-1} = \sqrt{\overline{\alpha}_{t-1}} \underbrace{\frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t, t)}{\sqrt{\overline{\alpha}_t}}}_{\text{predicted } x_0} + \underbrace{\sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\cdot\hat{\epsilon}_\theta(x_t, t)}_{\text{direction pointing to } x_t} + \underbrace{\sigma_t \epsilon_t}_{\text{Gaussian noise}} \quad , \epsilon_t \sim \mathcal{N}(0, I)$$

Denoising Diffusion Implicit Models (DDIM)

- DDIM uses $\sigma_t = 0, \forall t \geq 0$ in the generalized process.
- We can sample using the equation

$$x_{t-1} = \sqrt{\overline{\alpha}_{t-1}} \underbrace{\frac{x_t - \sqrt{1-\overline{\alpha}_t}\,\hat{\epsilon}_\theta(x_t, t)}{\sqrt{\overline{\alpha}_t}}}_{\text{predicted } x_0} + \underbrace{\sqrt{1-\overline{\alpha}_{t-1}-\sigma_t^2}\cdot\hat{\epsilon}_\theta(x_t, t)}_{\text{direction pointing to } x_t}$$

- This equation provides deterministic sampling.
- Faster sampling: Since the reverse trajectory is deterministic, we can skip steps!
- Consider the full reverse trajectory $x_T \rightarrow x_{T-1} \rightarrow \cdots \rightarrow x_0$.
- DDIM uses a decreasing sequence $\{\tau_1, \ldots, \tau_{S-2}\}$ to subsample a trajectory of length S

$$x_T \rightarrow x_{\tau_1} \rightarrow \cdots \rightarrow x_{\tau_{S-2}} \rightarrow x_0$$

- In practice, $S \ll T$ and in this way we can obtain faster sampling!

Sample Efficiency of DDIM

- DDIM with only $S = 10$ steps of reverse process achieves better FID score than DDPM with 1000 steps in the reverse process.

η: noise added at each step of the reverse process

		CIFAR10 (32 × 32)					CelebA (64 × 64)				
	S	10	20	50	100	1000	10	20	50	100	1000
DDIM	0.0	**13.36**	**6.84**	**4.67**	**4.16**	4.04	**17.33**	**13.73**	**9.17**	**6.53**	3.51
	0.2	14.04	7.11	4.77	4.25	4.09	17.66	14.11	9.51	6.79	3.64
η	0.5	16.66	8.35	5.25	4.46	4.29	19.86	16.06	11.01	8.09	4.28
DDPM	1.0	41.07	18.36	8.01	5.78	4.73	33.12	26.03	18.48	13.93	5.98

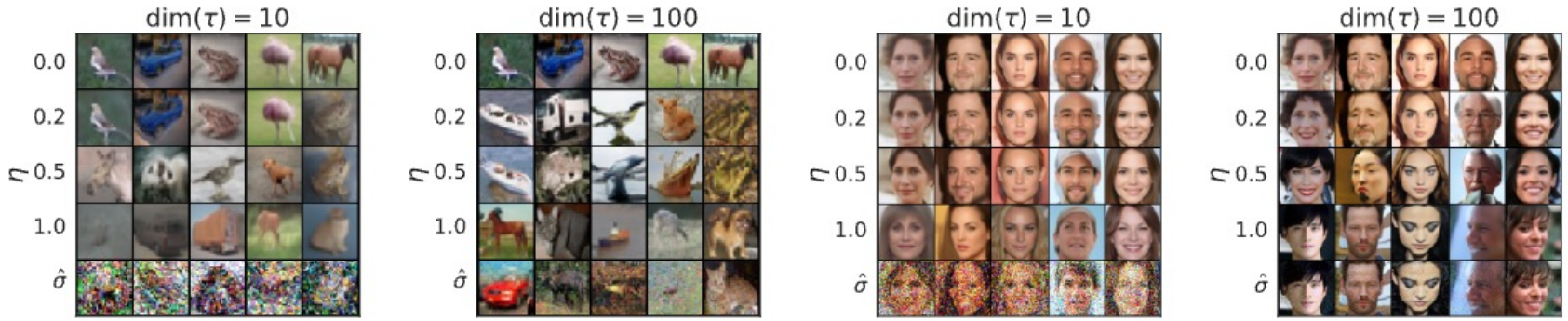


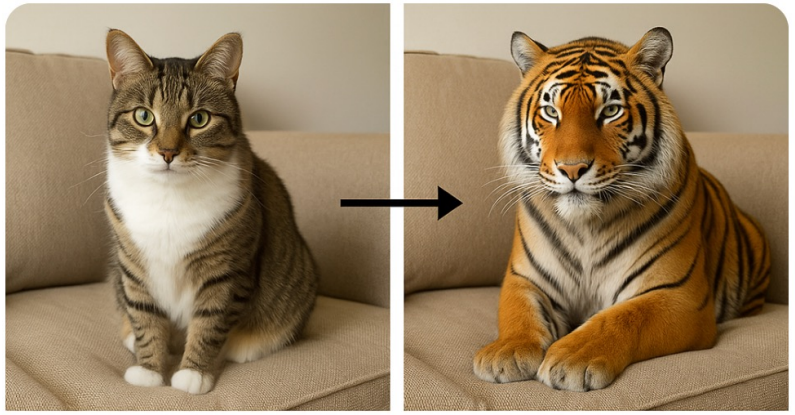
Figure 3: CIFAR10 and CelebA samples with $\dim(\tau) = 10$ and $\dim(\tau) = 100$.

What have we achieved so far?

- For image editing, we required exact inversion of the diffusion model and fast sampling.
- DDIM with $\sigma_t = 0$ provides deterministic sampling in a few steps.

- To perform image editing with DDIM:
  1. **Encode**: Run the forward process to get x_t for some intermediate t (partial noising of x_0).
  2. **Edit**: Modify the conditioning input.
  3. **Decode**: Run the reverse DDIM process using the new conditioning to get the modified image.

Input: "A photo of a **cat** on a couch"

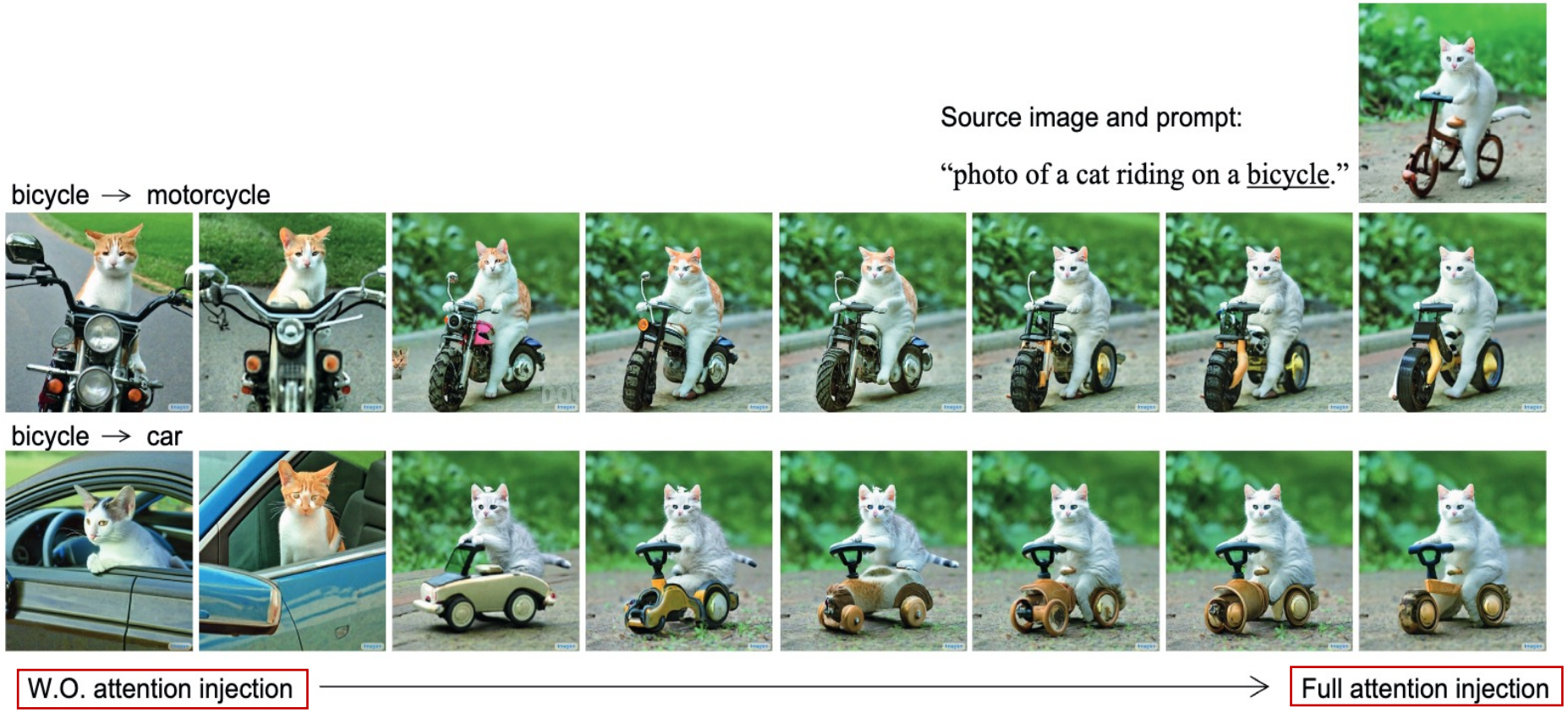


Edit: "A photo of a **tiger** on a couch"

- Next, we will see how to perform even more advanced edits in this space.
  - One example: Prompt2Prompt (P2P)

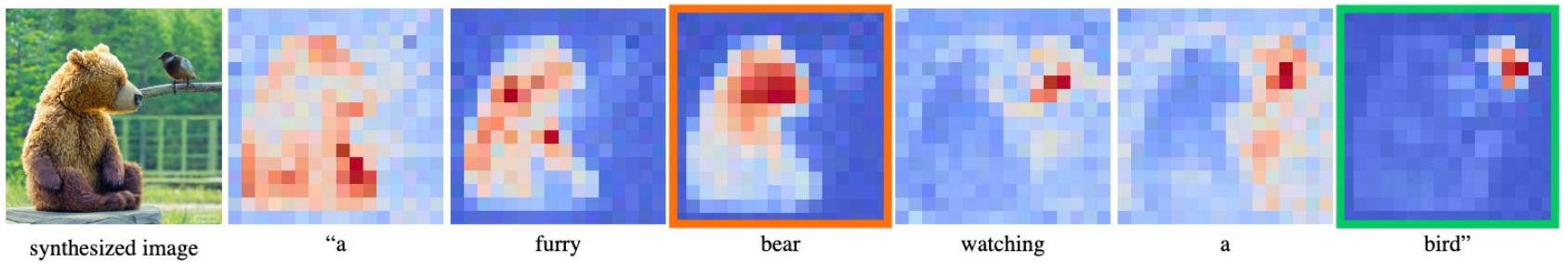
Prompt2Prompt

- DDIM Inversion has no symbolic (rigid) control for structural consistency.
- Prompt2Prompt (P2P) proposes to save the cross-attention maps during the forward process, edit the image and reuse the same attention maps during the reverse process.

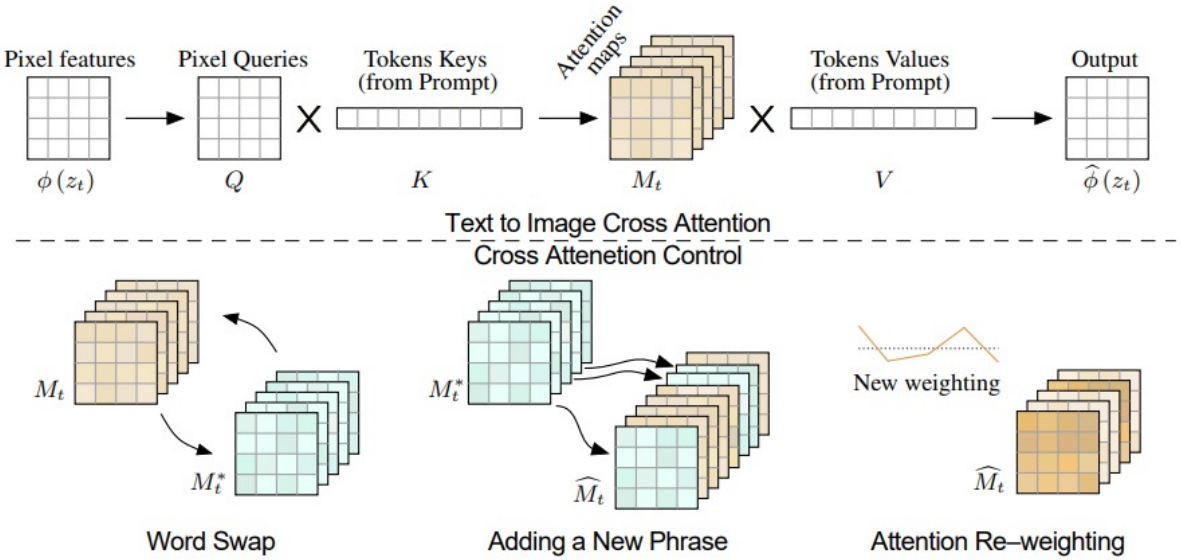


Prompt2Prompt

- The spatial layout of the generated image depends on the cross-attention maps, each indicating how each spatial location in the image attends to each word in the text.



- To perform image editing with P2P:
  1. **Save** the initial attention maps of the forward DDIM process.
  2. **Edit:** Compute the attention maps corresponding to the edit prompt.
  3. **Decode:** Inject the edited attention maps and run the reverse process.



How to perform cross-attention replacement?

- At each denoising step t and cross-attention layer ℓ, the UNet produces an attention map $M^{(\ell,t)} \in \mathbb{R}^{(H \times W) \times K}$, where K is the number of tokens.

Example

- Source Prompt P = "a **cat** on a couch" → Edit Prompt P' = "a **tiger** on a couch"
- **Lock/shared tokens**: keep their spatial maps from the source
- **Swap tokens**: copy the old token's map into the new token's column.
- For $i_{cat} \in P, j_{tiger} \in P'$ and S being the set of shared tokens, the attention map $M'^{(\ell,t)}$ corresponding to the edit prompt P' is given

$$M'^{(\ell,t)}[:, j] \leftarrow M^{(\ell,t)}[:, i], \text{ for } (i, j) \in S \textbf{ (copy)}$$

$$M'^{(\ell,t)}[:, j_{tiger}] \leftarrow M^{(\ell,t)}[:, i_{cat}] \textbf{ (swap)}$$

Samples of Edited Images

"Photo of a cat riding on a bicycle."

source image

cat → dog

"A photo of a butterfly on a flower."

source image

"...on a spikey flower."

"Photo of a house with a flag on a mountain."

source image

house → hotel

house → tent

house → car

house → tree

Conclusion on Image Editing

- The latent space of diffusion models can be used for image editing.
- Image editing using DDPM faces the problems of inversion and slow sampling.
- To speed up the sampling process, we considered a generalized non-Markovian forward process.
- DDIM provides deterministic reverse process and fast sampling.
- Prompt2Prompt allows for edits in the image, while maintaining the structural properties of the initial image.